MTH 287 Project, Spring 2016 (due Wednesday July 27, 2016, 4:30pm)

Description: A discrete mathematics course teaches more than a set of skills and mathematical facts. It teaches the scientific way of thinking, helping students to think logically and analytically, employing mathematical reasoning to analyze a given problem, synthesize the available knowledge, consider the alternative solutions, and finally produce a well-written and clearly explained solution.

In this assignment, you should provide the detailed analysis of six problems – one from each of the six groups below. Each problem should be on its own paper, labeled with your name and the problem number (Ex: I.2.) at the top. There will be six different papers turned in for this project.

You should correctly identify all concepts pertinent to the subject matter; clearly explain step-by-step solution or proof; and provide examples illustrating each concept. Your paper must be original – it may be run through *SafeAssign* to determine authenticity. Plagiarism means zero points. All authors, works, and websites must be appropriately cited. A production of College-level paper may require the use of multiple resources such as various textbooks and websites. Please use suggested sites in addition to your textbook and other online resources. Wikipedia is not in itself a good site for a citation, but can lead you to good sites.

Your work must show the level of knowledge and mathematical sophistication, as well as writing skills, appropriate for a transfer student majoring in Mathematics, Science, Computer Science, or other fields. Above all, your critical thinking and analytical abilities must demonstrate the potential for higher-level University studies. Only the essay page must be typed with appropriate citations. The others may be neatly handwritten and need citation only if materials outside of the class and the text are used.

Each problem will be worth $16\frac{2}{3}$ points. The total for the project is 100 points. If you turn your project (or some pages of it) in early, you may make improvements or corrections and resubmit it. The final due date is the date listed above. The final project will be evaluated by at least one person who is not your instructor and the original will be kept in your student folder along with items collected from other courses.

No projects will be accepted late.

I. Read & Understand…

1. Let A be the set of non-negative integers. Define relation R on A by (a, b)R(c, d) iff a + d = b + c. Prove that R is an equivalence relation.

2. Let A = {1, 2, 3, …, 15}. Consider the relation R on set A such that (a, b)R(c, d) iff a + d = b + c. Find the equivalence class of (2, 7) and the equivalence class of (3,4). Explain all steps.

3. Consider a relation of division, R, on the set of positive integers: (a, b) is in R iff a|b. Determine if relation R is reflexive, symmetric, antisymmetric, or transitive. Is it an equivalence relation. Explain in detail.

II. Determine appropriate techniques…

1. Consider the function $f(x) = 2^x$. Does f(x) have an inverse? Explain in detail how you know. If it does have an inverse, find it. Include graphs in your explanation.

2. Let f(x) = (2x – 3)/(5x – 7). Find the inverse of f(x) and express the domain and range of $f(x)^{-1}$ in the interval notation or set-builder notation. Include a discussion of the relationship between the asymptotes in f(x) and its inverse.

3. Prove/disprove that function f(x) = $2x^2$ – 3 has an inverse *without* finding the inverse. If it does not have an inverse, indicate a restriction (on its domain) that would have an inverse.

III. Determine appropriate equation or steps…

1. Prove that 1 + 4 + 7 + … (3n – 2) = n(3n – 1)/2 for n ≥ 2.

2. Prove $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\cdots+\frac{1}{\sqrt{n}} \geq \sqrt{n}$ for n ≥ 1

3. Prove 4 | $(3^{2n-1}+1)$ for any integer ≥ 1

IV. Appropriate technology…

1. Write a computer program (possibly in Java), or for your graphing calculator to calculate first 20 terms of a recursively defined sequence, where the first term is equal to 8 and each consecutive term is equal to 6 less than twice the previous term. Show both the code and the output.

2. Use graphing calculator to find intersection(s) of $f(x) = \ln(x^2 - 1)$ and $g(x) = (4 - x^2)^{1/2}$. Explain each step in detail. Give all of the solutions

3. Use graphing calculator to find all zeros of polynomial $p(x) = 2x^3 - 3x^2 - 4x - 5$. Explain each step in detail. Give all of the solutions

V. Evaluate reasonability…

1. Let A = {1, 2, 3, 4, 5, 6}, and let R be the relation on A defined "x divides y" for all x, y in A. Write the relation R as a set of ordered pairs (x, y). Draw a digraph for the relation. Is it an equivalence or partial order relation? Explain your solution.

2. Determine the inverse relation R^{-1} of R in problem 1. Can you describe R^{-1}as "y is a multiple of x"? Explain why or why not. Is it an equivalence or partial order relation?

3. Explain in detail the validity/invalidity of the argument

All mathematicians are interesting people.
Only uninteresting people become politicians.
Every genius is a mathematician.
Therefore, some geniuses are politicians.

VI. Writing…
Write a short essay explaining one of the following:

1. Giuzeppe Peano and Peano axioms.
2. Gregor Cantor and Cantor paradox.
3. Bertrand Russel and Russel paradox.
4. Henri Poincare and his view on transfinite numbers.
5. Kurt Godel and Incompleteness Theorem